Handling hybrid and missing data in constraint-based causal discovery to study the etiology of ADHD

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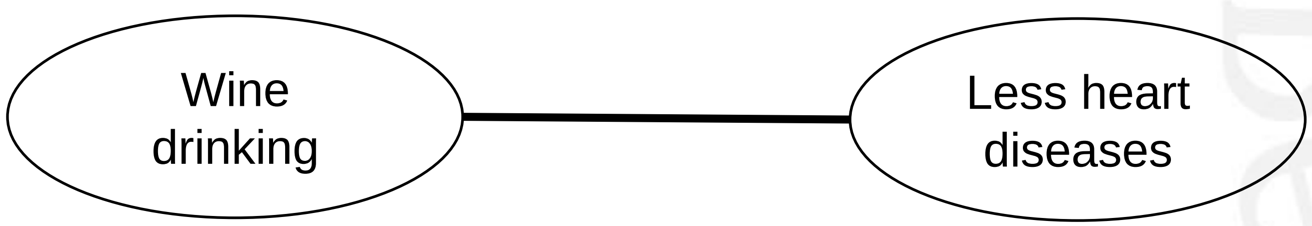
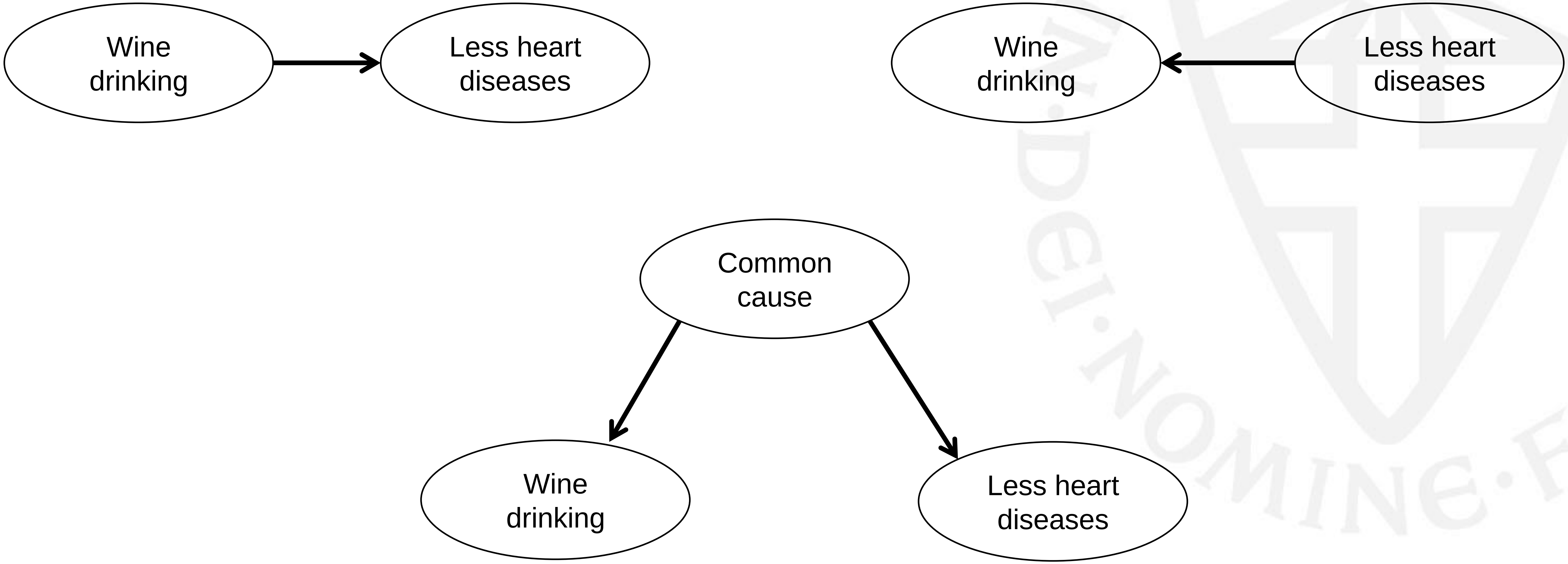
Does wine drinking prevent heart disease?

Wine drinking and lower rate of heart disease are associated



Does wine drinking prevent heart disease?

All possible models



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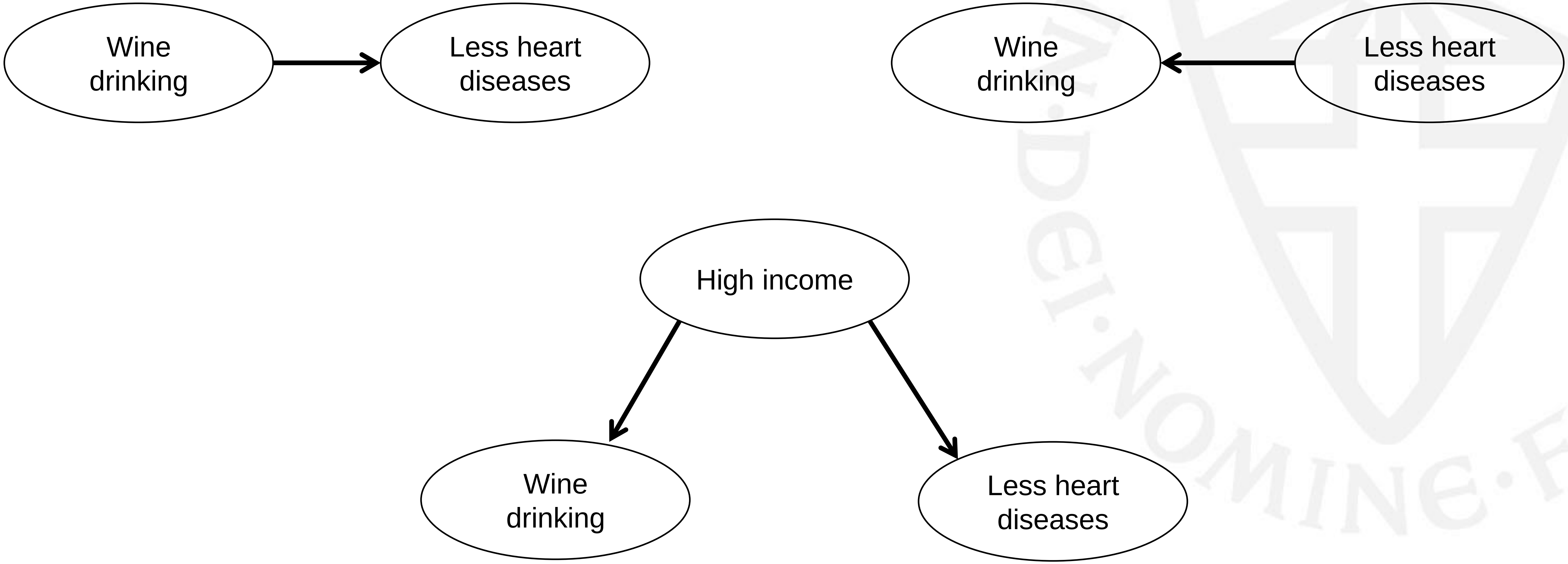


A way to learn causality

1. Take randomly 200 people
2. Randomly split them in **controls** and **treatment** groups
3. Force **treatment** group to drink wine, forbid **control** group to drink wine
4. Wait 40 years
5. Measure correlation

[Randomized Controlled Trial]

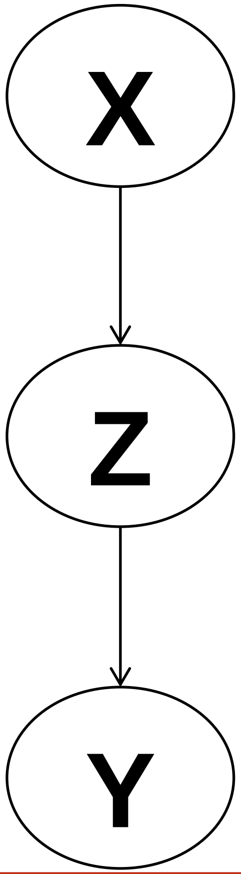
Can we learn causal relationships from observed data?

Yes!

Conditional Independence

X and ***Y*** are conditionally independent given ***Z*** :
Given ***Z***

- knowledge of ***X*** provides no information for ***Y***
- knowledge of ***Y*** provides no information for ***X***



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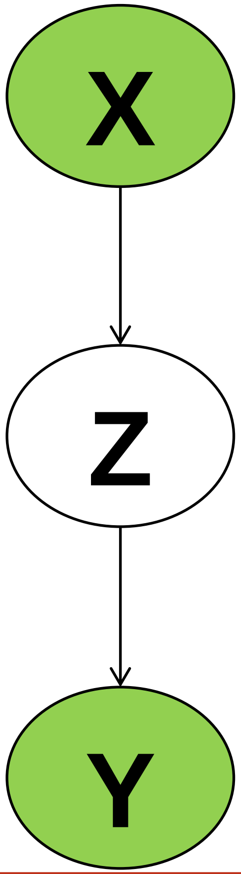

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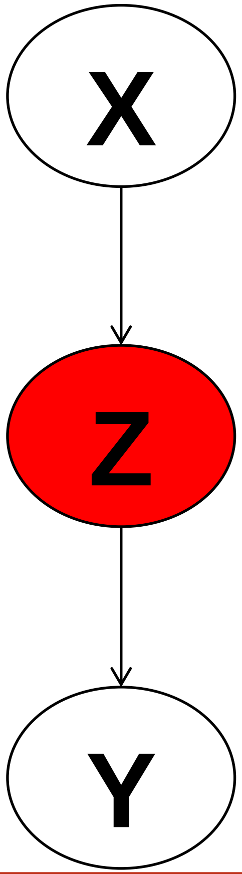


Learning causal network

Bayesian constraint-based causal discovery:

- Uses Bayesian approach to estimate the reliability of the causal statements, avoiding propagation of unreliable decisions

T. Claassen, T. Heskes. **A Bayesian approach to constraint based causal inference**. In *UAI 2012*

BCCD

Basic idea:

- **Step 0** Start with a fully connected graph.
- **Step 1** Estimate the reliability of a causal statement $(X \rightarrow Y)$ using Bayesian score.
- **Step 2** If a causal statement declares a variable conditionally independent, delete an edge.
- **Step 3** Rank all causal statements and orient edges in the graph.

BCCD

The reliability of the causal statement L given the data **D** using Bayesian score**:**

$$p(L|D) = \frac{\sum_{\mathcal{M} \in M(L)} p(D|\mathcal{M})p(\mathcal{M})}{\sum_{\mathcal{M} \in M} p(D|\mathcal{M})p(\mathcal{M})}$$

There is a closed form solution for $p(D|\mathcal{M})$:

- Discrete random variables - BD metric
- Continuous Gaussian variables - BGe metric

BCCD

Advantages of the method:

- Robust
- Can handle latent variables
- Gives an indication whether an edge does exist or not

Limitation of the method:

- Works only with discrete variables or Gaussian variables
- Cannot handle missing values

Undirected graphs

- Precision matrix- inverse of correlation matrix
- Precision matrix - the set of conditional independencies
- Add sparsity constraints

Undirected graphs

Glasso to find optimum

$$\Theta_\lambda = \text{argmax}_\Theta \{\underbrace{\text{logdet}(\Theta) - \text{tr}(\Theta S)}_{\text{Goodness of fit}} - \underbrace{\lambda\|\Theta\|_1}_{\text{Sparsity penalty}}\}$$

- $\Theta = \Sigma^{-1}$ inverse of correlation matrix
- S- empirical correlation matrix

- Spearman instead of Pearson partial correlation
- Adjust Spearman correlation, to make it closer to Pearson
- Shift correlation matrix to the closest one if it is negative definite
- Use EM if there are missing values

Assumptions

- Data is a mixture of discrete and continuous variables
- Data is missing completely at random (MCR)
- Relationships between variables are monotonic, i.e. variables follow a so-called non paranormal distribution

Method extension

- BIC score:

$$BIC\ score(\boldsymbol{D}|\mathcal{G}) = \underbrace{M\sum_{i=1}^{n} I(X_i, Pa_{X_i})}_{\text{Goodness of fit}} - \underbrace{\frac{\log M}{2}\mathrm{Dim}[\mathcal{G}]}_{\text{Complexity penalty}}$$

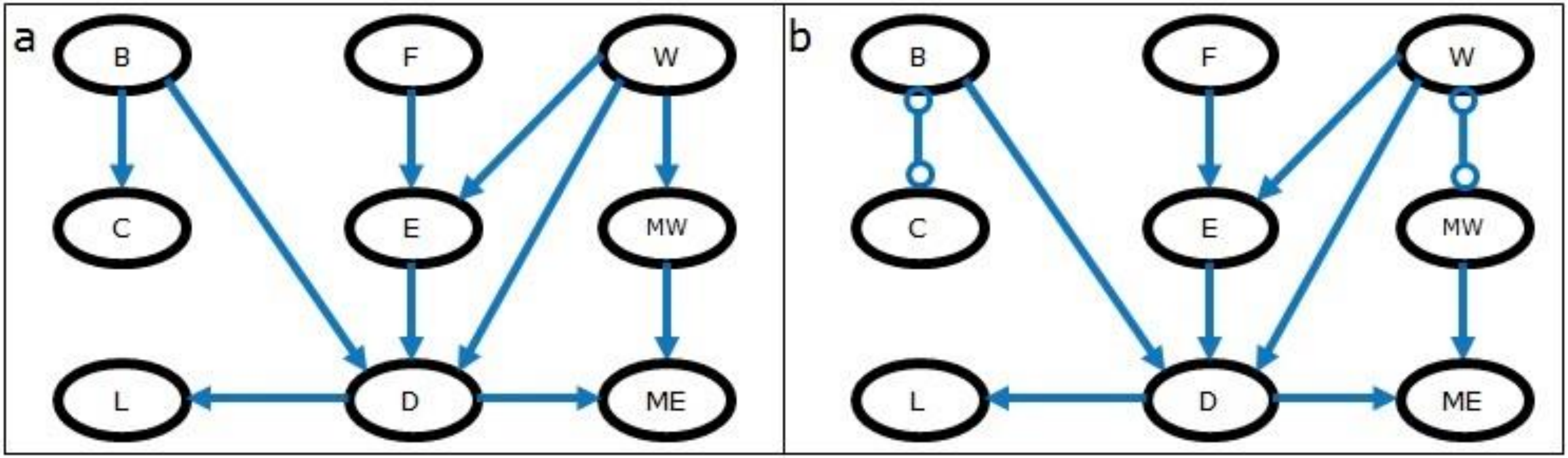
- Mutual information

$$I(x_1, \dots, x_n) = -\frac{1}{2}\log\frac{|R|}{|R_{Pa_i}|}$$

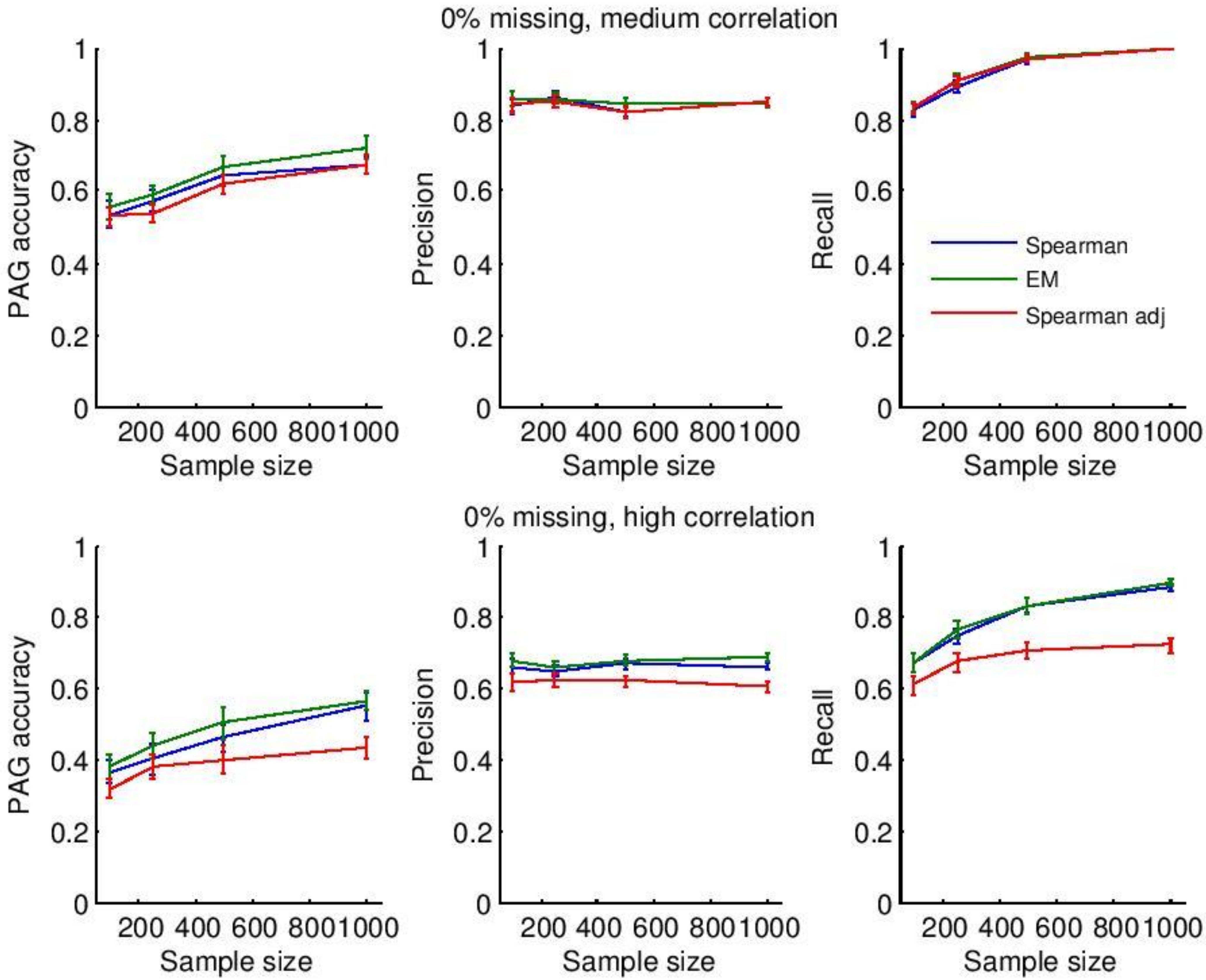
- Use Spearman instead of Pearson
- Use EM if there are missing values

Simulated data

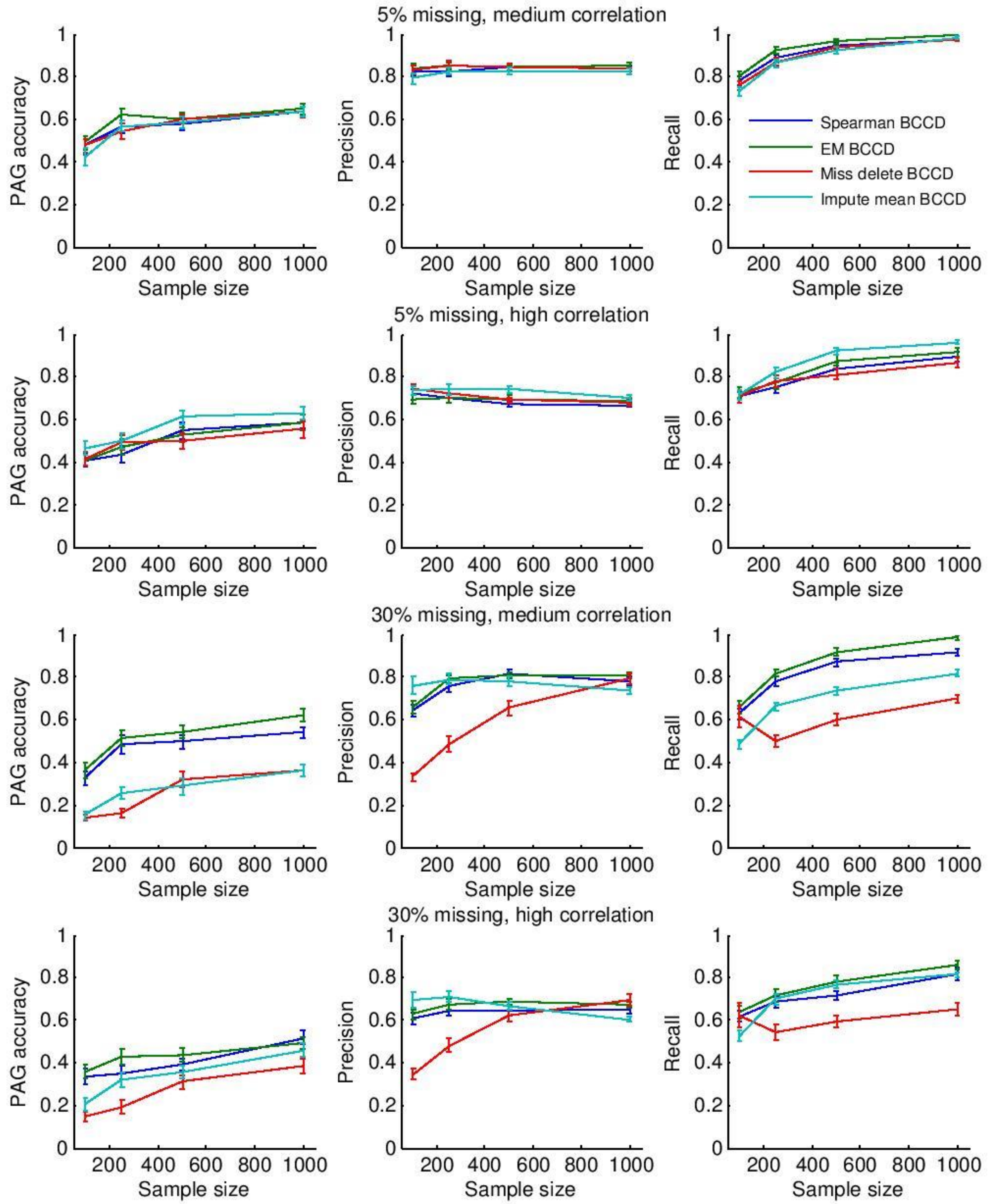
- Waste Incinerator Network, x^3 transformed
- Sample size: 100, 250, 500, 1000
- Estimated PAG accuracy, precision, and recall



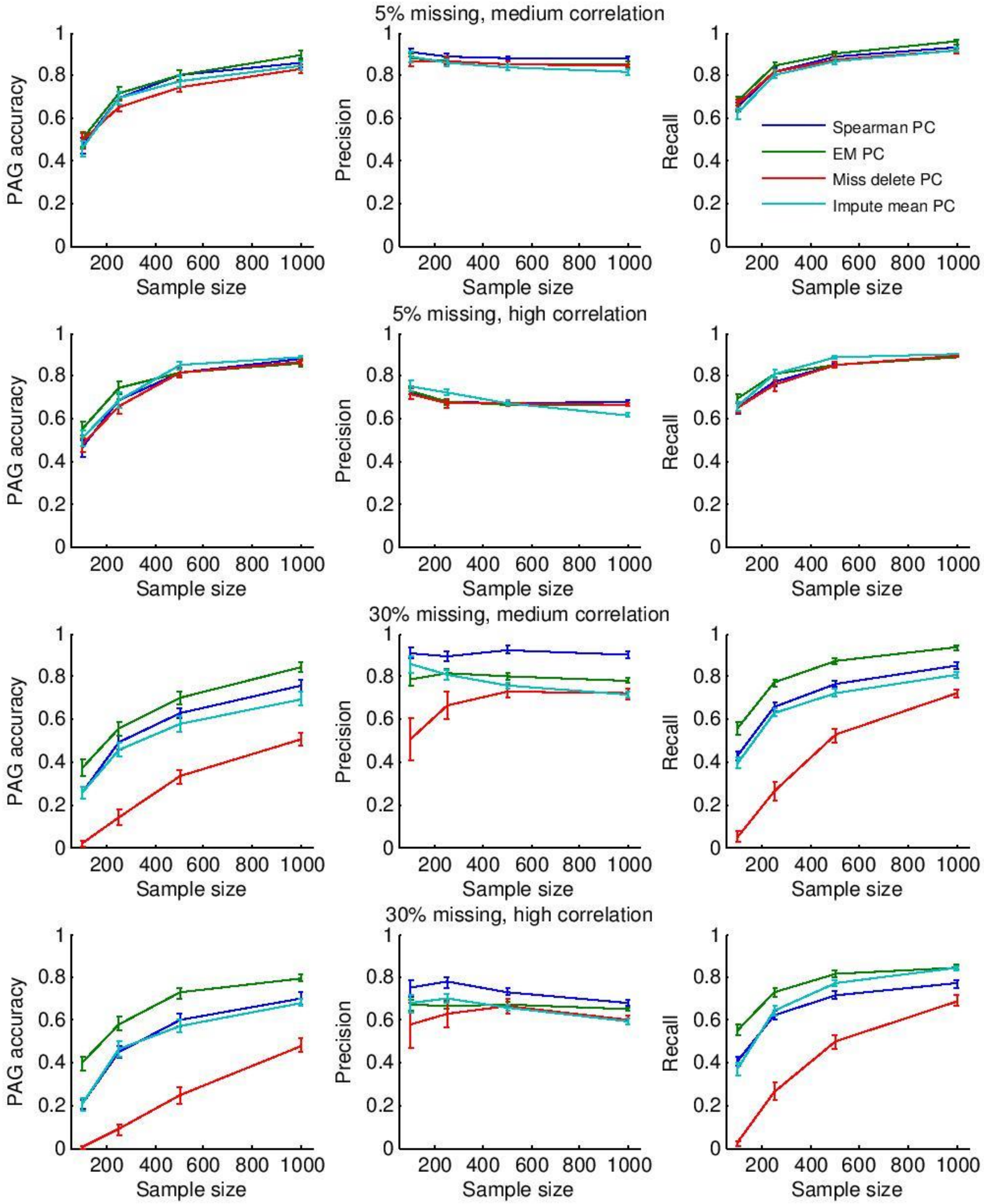
0% missing


0% missing, medium correlation
PAG accuracy
Precision
Recall
Sample size
200 400 600 800 1000
0 0.2 0.4 0.6 0.8 1
Spearman
EM
Spearman adj
0% missing, high correlation

5%, 30% missing BCCD



5%, 30% missing PC



Conclusions

- EM performs better than other methods when there is a significant amount of missing values
- Spearman adjusted leads to unstable matrix and many spurious edges

Real world Data set, ADHD MID task

Type of data:

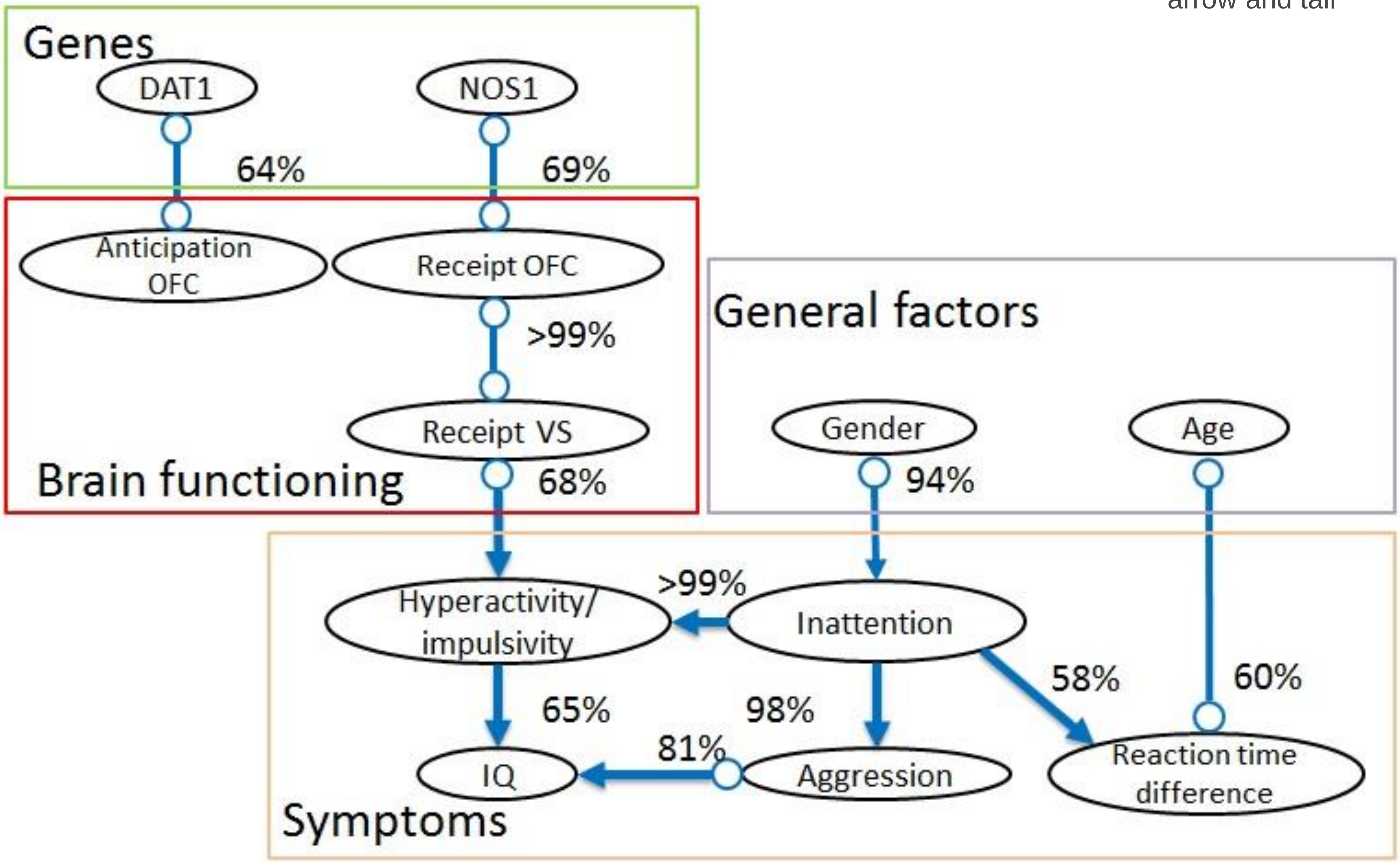
- Genetic information (NOS1, DAT1)
- Brain activation (OFC, VS, anticipation and feedback)
- Behavioral (symptoms, aggression, reaction time, IQ)
- General (age, gender)

Assumptions

- Assumed that missing values are missing at random
- Combined two types of symptoms assessments: by parents and by psychiatrist.
- Incorporated prior knowledge that nothing can cause:
    - Gender
    - Feedback VS is not caused by HI

Real world data ADHD MID task

A → **B**: A causes B
A ↔ **B**: latent common cause
A — **B**: selection bias
O: cannot distinguish between arrow and tail



Real world Data set, ADHD reversal task

Type of data:

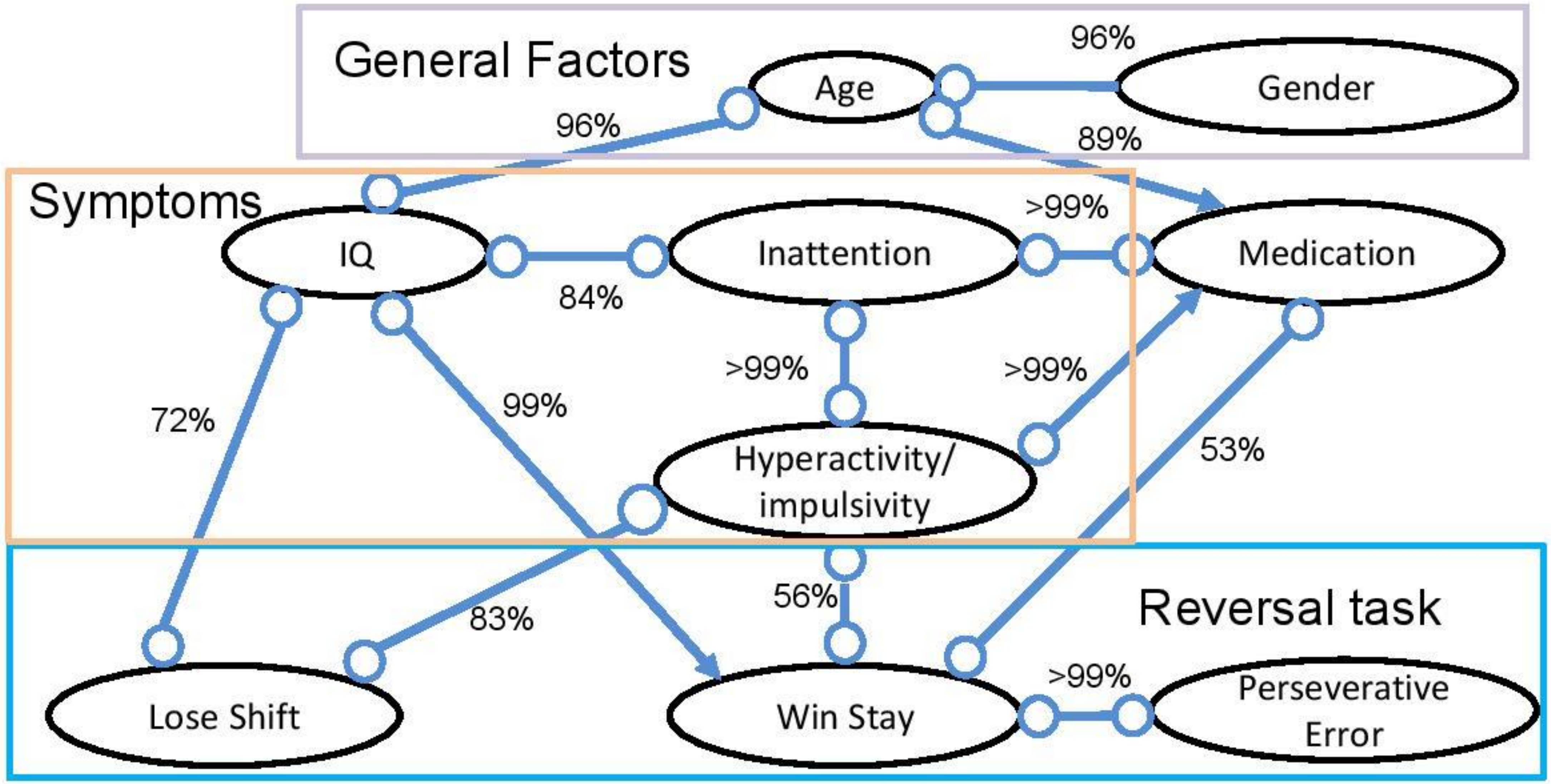
- Experiment related (lose shift, win stay, error)
- Behavioral (symptoms, IQ)
- General (age, gender)

Assumptions

- Assumed that missing values are missing at random
- Incorporated prior knowledge that nothing can cause:
    - Gender

Real world data ADHD reversal task

A → B: A causes B
A ↔ B: latent common cause
A — B: selection bias
O: cannot distinguish between arrow and tail



Conclusions and Future work

- Extension of the BCCD algorithm for mixtures of discrete and continuous variables

- Works well under the assumption of non paranormal data and values MAR

- Further developments:
  - More complex relationships
  - Longitudinal data

Thank you for your attention!